Appendix A
Other Special Functions

Mathcad sometimes returns a symbolic expression in terms of a function that isn't one of Mathcad's built-in functions.

You can define many of these functions in Mathcad. See the “Other Special Functions” topic in the QuickSheets of the Resource Center for examples.

The list below gives definitions for these functions. Except for Ei, erf, and Zeta, all of which involve infinite sums, and also W, you can use such definitions to calculate numerical values in Mathcad.

Function Definitions

Name	**Definition**
Euler's constant	$\gamma = \lim_{n \to \infty} \left(\sum_{k=1}^{n} \frac{1}{k} - \ln(n) \right) = 0.57721566\ldots$
Hyperbolic cosine integral	$\mathrm{Chi}(x) = \gamma + \ln(x) + \int_0^x \frac{\cosh(t) - 1}{t} dt$
Cosine integral	$\mathrm{Ci}(x) = \gamma + \ln(x) + \int_0^x \frac{\cos(t) - 1}{t} dt$
Dilogarithm function	$\mathrm{dilog}(\mathrm{x}) = \int_1^x \frac{\ln(t)}{1-t} dt$
Dirac delta (unit impulse) function	$\mathrm{Dirac}(x) = 0$ if x is not zero. $\int_{-\infty}^{\infty} \mathrm{Dirac}(x) dx = 1$
Exponential integral	$\mathrm{Ei}(x) = \gamma + \ln(x) + \sum_{n=1}^{\infty} \frac{x^n}{n \cdot n!}$ $(x > 0)$
Complex error function	$\mathrm{erf}(z) = \frac{2}{\sqrt{\pi}} \sum_{n=0}^{\infty} \frac{(-1)^n z^{2n+1}}{n!(2n+1)}$ (for complex z)
Fresnel cosine integral	$\mathrm{FresnelC}(x) = \int_0^x \cos\left(\frac{\pi}{2} t^2\right) dt$

Fresnel sine integral	$\mathrm{FresnelS}(x) = \int_0^x \sin\left(\frac{\pi}{2}t^2\right)dt$
Incomplete elliptic integral of the second kind	$\mathrm{LegendreE}(x, k) = \int_0^x \left(\frac{1 - k^2 \cdot t^2}{1 - t^2}\right)^{1/2} dt$
Complete elliptic integral of the second kind	$\mathrm{LegendreEc}(k) = \mathrm{LegendreE}(1, k)$
Associated complete elliptic integral of the second kind	$\mathrm{LegendreEc1}(k) = \mathrm{LegendreEc}(\sqrt{1 - k^2})$
Incomplete elliptic integral of the first kind	$\mathrm{LegendreF}(x, k) = \int_0^x \frac{1}{\sqrt{(1 - t^2)(1 - k^2 \cdot t^2)}} dt$
Complete elliptic integral of the first kind	$\mathrm{LegendreKc}(k) = \mathrm{LegendreF}(1, k)$
Associated complete elliptic integral of the first kind	$\mathrm{LegendreKc1}(k) = \mathrm{LegendreKc}(\sqrt{1 - k^2})$
Incomplete elliptic integral of the third kind	$\mathrm{LegendrePi}(x, n, k) = \int_0^x \frac{1}{\sqrt{(1 - n^2 \cdot t^2)\sqrt{(1 - t^2)(1 - k^2 \cdot t^2)}}} dt$
Complete elliptic integral of the third kind	$\mathrm{LegendrePic}(n, k) = \mathrm{LegendrePi}(1, n, k)$
Associated complete elliptic integral of the third kind	$\mathrm{LegendrePic1}(k) = \mathrm{LegendrePic}(n, \sqrt{1 - k^2})$
Digamma function	$\mathrm{Psi}(x) = \frac{d}{dx}\ln(\Gamma(x))$
Polygamma function	$\mathrm{Psi}(n, k) = \frac{d^n}{dx^n}\mathrm{Psi}(x)$
Hyperbolic sine integral	$\mathrm{Shi}(x) = \int_0^x \frac{\sinh(t)}{t} dt$

Sine integral

$$\mathrm{Si}(x) = \int_0^x \frac{\sin(t)}{t}\,dt$$

Lambert W function

$\mathrm{W}(x)$ is the principal branch of a function satisfying $\mathrm{W}(x) \cdot \exp(\mathrm{W}(x)) = x$.
$\mathrm{W}(n, x)$ is the *n*th branch of $\mathrm{W}(x)$.

Riemann Zeta function

$$\mathrm{Zeta}(x) = \sum_{n=1}^{\infty} \frac{1}{n^x} \quad (x > 1)$$

Comments

The Psi function and Γ appear frequently in the results of *indefinite* sums and products. If you use a single variable name rather than a full range in the index placeholder of a summation or product, and you choose **Evaluate Symbolically** or another symbolic evaluation command, Mathcad will attempt to calculate an indefinite sum or product of the expression in the main placeholder. The indefinite sum of $f(i)$ is an expression $S(i)$ for which $S(i+1) - S(i) = f(i)$.

The indefinite product of $f(i)$ is an expression $P(i)$ for which $\frac{P(i+1)}{P(i)} = f(i)$.