CHAPTER 3
OPTICAL INSTRUMENTS

3.1 *Introduction*

The title of this chapter is to some extent false advertising, because the instruments described are the instruments of first-year optics courses, not optical instruments of the real world of optical technology. Thus a telescope consists of a long focal length lens called the object glass and a short focal length lens called the eyepiece, and the magnification is equal to the ratio of the focal lengths. Someone whose experience with telescopes is limited to this concept of a telescope would scarcely recognize a real telescope. A real telescope would consist of an overwhelming mass of structural engineering intertwined with a bewildering array of electronics, wires and flashing lights. There would be no long focal length lens. Instead there would be a huge mirror probably with a hole in the middle of it. There would be no eyepiece, nor anyone to look through it. The observer would be sitting in front of a computer terminal, quite possibly in another continent thousands of miles away.

Thus the intent of the chapter is mainly to give a little bit of help to beginning students who are struggling to answer examination question of the type "A microscope consists of two lenses of such-and-such focal lengths. What is the magnification?"

None of this means, however, that the simple and fundamental principles described in this chapter do not apply to real instruments. They most certainly do apply. This is just a beginning.

3.2 *The Driving Mirror*

The mirror inside a car above the driver's head and the outside mirror on the driver's side are usually plane mirrors. The mirror I have in mind for this section, however, is the outside mirror on the passenger's side. This is usually a convex mirror with some words inscribed on it that say something like "OBJECTS IN MIRROR ARE CLOSER THAN THEY APPEAR".

The image formed by the convex mirror is actually an erect, diminished, virtual image, and it "appears" just a few inches behind the surface of the mirror. The object is <u>much further</u> away that it "appears" to be!

That, however, is not the main purpose of discussing this important scientific instrument. The reason that the outside mirror on the passenger side is convex is to give the driver a large *field of view*, so that this gives us an opportunity to think about the *field of view* of an optical system.

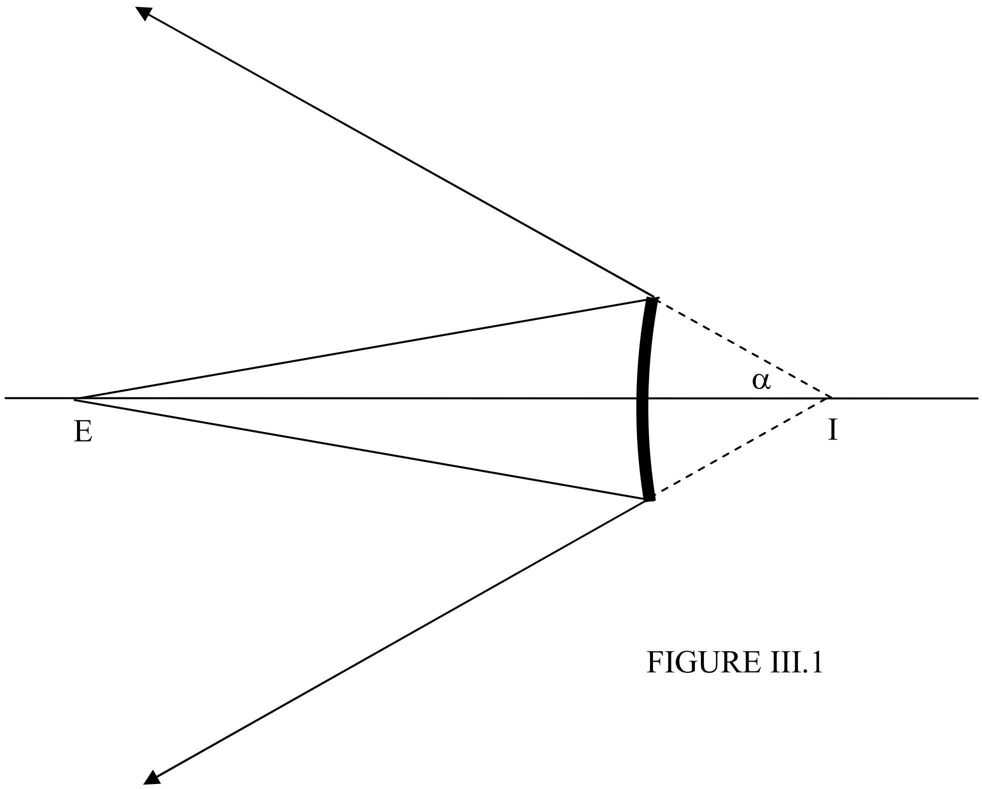

FIGURE III.1

In figure III.1, we see a convex lens, and the observer's eye is at E. (As with previous chapters, angles are supposed to be small, my artistic efforts notwithstanding.) The angle α is evidently the *radius of the field of view.* How do we calculate it? Well, I hope it is clear from the drawing that the point I is actually the virtual *image of the eye* formed by the mirror. That being so, we can say:

The angular size of the field of view is equal to the angle subtended by the mirror *<u>at the image of the eye.</u>*

This is true of a concave mirror as well as of a convex or indeed a plane mirror, and is equally true when we look through a lens. (Draw the corresponding diagrams to convince yourself of this.)

Example. Your eye is 50 cm in front of a convex mirror whose diameter is 4 cm and whose radius of curvature is 150 cm. What is the angular diameter of the field of view?

First we need to find the position of the image of the eye. Suppose it is at a distance q behind the mirror.

Final convergence = Initial convergence + power

The light is diverging before and after reflection, so both convergences are negative. The power of a mirror is $-2n/r$, and here $n = 1$ and $r = +150$ cm, because the surface is convex. Thus

$$-\frac{1}{q} = -\frac{1}{50} + \frac{-2}{+150},$$

so the image is 30 cm behind the mirror. The diameter of the mirror is 4 cm, so that angular diameter of the mirror from I (i.e. the field of view) is 4/30 = 0.1333 rad = 7° 38'.

3.3 *The Magnifying Glass*

Two points about a magnifying glass to begin with. First, apparently rather few people understand how to use this complicated scientific instrument. The correct way to use it is to *hold it as close to your eye as possible*. The second point it that it doesn't magnify at all. The angular size of the image is exactly the same as the angular size of the object.

Before examining the magnifying glass, it is probably useful to understand just a little about the workings of the human eye. I am not a biologist, I am very squeamish about any discussion of eyes, so I'll keep this as basic as possible. When light enters the front surface or *cornea* of the eye, it is refracted in order to come to a focus on the back surface of the *retina.* The image on the retina is a real, inverted image, but the brain somehow corrects for that, so that objects look the right way up. While most of the refraction takes place at the cornea, some adjustment in the effective focal length is made possible by a flexible lens, whose power can be adjusted by means of *ciliary muscles.* The adjustment of this lens enables us to *accommodate* or bring to a focus objects that are at varying distances from us.

For an eye in good condition in a young person, the eye and the ciliary muscles are most relaxed when the eye is set to bring to a focus light from an infinitely-distant object – that is, when the eye is set to receive and bring to a focus light that is parallel before it enters the eye. In order to focus on a nearby object, the ciliary muscles have to make a bit of an effort to increase the power of the lens. They can increase the power of the lens only so far, however, and most people cannot focus on an object that is closer than a certain distance known as the *near point.* For young people the near point is usually taken to be 10 inches or 25 cm in calculations. The actual real point may differ from person to person; the figure of 25 cm is a "standard" near point. With older people, the near point recedes, so that 25 cm is too close for comfort, and the lens becomes less flexible.

When we use a magnifying glass properly (by holding it very close to the eye) we automatically place it so that the object we are looking at is at the focal point of the lens, and consequently parallel light emerges from the lens before it enters our eye. We don't think about this. It is just that the ciliary muscles of the eye are most relaxed when they are set to bring to bring parallel light to a focus. It is merely the most comfortable thing to do. Figure III.2 shows a magnifying glass at work. As usual, angles are small and the lens is thin.

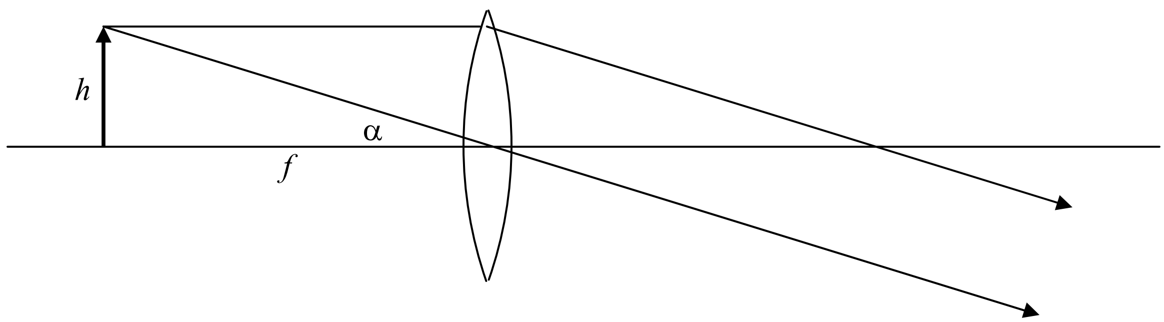

FIGURE III.2

The object is in the focal plane of the lens. I draw two rays from the tip of the object. One is parallel to the axis, and, after passing through the lens, it passes through the focus on the other side of the lens. The other goes through the centre of the lens. (Since the lens is thin, this ray is not laterally displaced.) Parallel rays emerge from the lens. The eye is immediately to the right of the lens, and it easily brings the parallel rays to a focus on the retina.

Although the lens does not actually produce an image, it is sometimes said that the lens produces "a virtual image at infinity". The angular size of this virtual image is α, which is also the angular size of the object, namely $\alpha = h/f$. Thus the angular size of the image is the same as the angular size of the object, and the lens hasn't magnified at all!

However, if you put the object at a distance f (perhaps a few cm) from the eye without using the lens, you simply couldn't focus your eye on it. Without the lens, the closest that you can put the object to your eye would be D, the distance to the near point - 25 cm for a young eye. The angular size of the object would then be only h/D.

The angular magnification of a magnifying glass is therefore *defined* as

$$\frac{\text{angular size of the image (which is } h/f\text{)}}{\text{angular size of the object when the object is at the near point (which is } h/D\text{)}}\,.$$

Hence the magnification is equal to D/f. The near point is taken to be 25 cm, so that a lens of focal length 2.5 cm has an angular magnification of 10.

If you bring the object just a little inside the focal plane, the light emerging on the other side will diverge, as it were from a virtual image that is no longer at infinity. (Figure III.3).

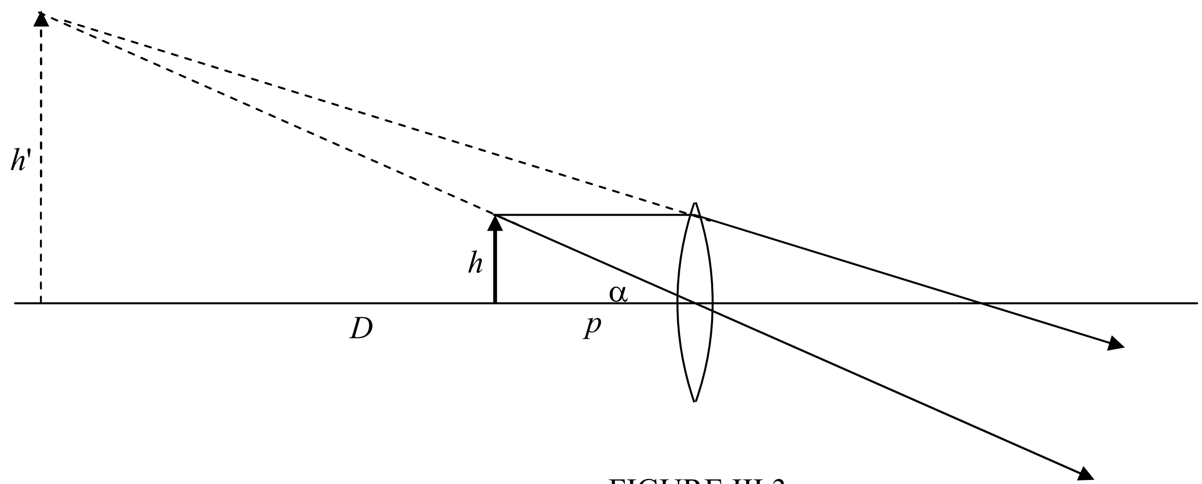

FIGURE III.3

There is no point, however, in bringing the image closer than the near point. If you bring it to the near point, what must the object distance p be? A simple lens calculation shows that $p = \frac{fD}{f + D}\cdot$ The angular size of the image is therefore $\frac{h(f + D)}{fD}\cdot$ Since the angular size of the object when the object is at the near point is h/D, the angular magnification is now $\frac{D}{f} + 1$ when the image is at the near point. This, for our $f = 2.5$ cm lens, the angular magnification is then 11.

3.4 *Spectacle Lenses*

The less time I spend thinking about eyes the better. However, for a number of different reasons it may happen that, when parallel light enters the relaxed eye, it may be brought to a focus *before* the retina. In effect the lens, or the cornea, of the eye is too strong, or perhaps the eyeball is too deep. It is easy to see objects that are close up, but light from more distant objects is brought to a focus too soon. The eye is said to be *myopic* or *short-sighted* or *near-sighted.* All that is needed is a weak diverging lens in front of the eye.

Perhaps when parallel light enters the eye, it is brought to a focus *behind* the retina. Maybe the lens or the cornea is too weak, or the eyeball isn't deep enough. By contracting the ciliary muscles you can bring parallel light to a focus, and may even be able to focus on distant objects. But you just cannot focus on nearby objects. Your near point is much more distant than the standard 25 cm. In that case the eye is *hypermetropic*, or *long-sighted* or *far-sighted.* It is easily corrected with a weak converging lens in front of the eye. It is normal for the near point to recede with age, and weak convex glasses are required. Such glasses to not "magnify"; they merely enable you to focus on objects that are closer than your near point – just as a so-called "magnifying glass" does. If you are hypermetropic, looking at large print won't help! Large print won't come to a focus any more than small print will.

Other eye defects, such as astigmatism, aren't so easily corrected with a simple lens, and require specially shaped (and expensive!) lenses.

3.5 *The Camera*

The camera is a box with a lens in one side of it and a photographic film or a CCD on the opposite side. The distance between camera lens and film can be changed so as to focus on objects at various distances. The aperture can also be changed. In dim light you need to open the aperture up to let a lot of light in; but this makes the image less sharp, and you have a smaller depth of field.

The *aperture* of a lens is merely its diameter, and it is usually expressed as a fraction of the focal length. Thus an aperture of $f/22$ is a small aperture. You can use this only in strong light, but you will then have nice sharp images and a large depth of field. An aperture of $f/6.3$ is wide open; the cone of light inside the camera is quite steep, and focussing is then quite critical. You use such a wide aperture only if you are forced to by dim light. The apertures typically available on a camera are often in steps with a ratio of approximately $\sqrt{2}$ from one to the next. As you increase the aperture by a factor of $\sqrt{2}$, you get twice as much light on the film (because this depends on the area of the exposed lens), so presumably you can cut the exposure time by one half. This is probably true for a CCD camera; the degree of blackening of a photographic film is not quite proportional to the product of the illuminance and the time, but at least it serves as a rough guide.

How is the *depth of focus* related to the aperture? Let us suppose that we have a lens that is free of aberrations such as spherical aberration, and that a point object produces a point image in the focal plane. If your film or CCD is not exactly in the plane, it will be illuminated not by a point image but by a small circle of finite diameter. If this circle is smaller than the grain or pixel size, you may wish to regard it as not seriously out of focus. So the question is: How far can you move the film away from the focal plane in either direction without the image being seriously out of focus? This range is the *depth of focus.*

In figure III.4 we see a cone of light converging from a lens of radius R to a focal point at distance f. Let us suppose that we place a film or CCD at the plane indicated by the dotted line at a distance x from the focal point, and that we are prepared to tolerate an out-of-focus "image" of radius r. From similar triangles we see that $x/r = f/R$. Or, if D is the diameter of the lens, and d is the diameter of the tolerable out-of-focus circle, x/d

= f/D. Thus we can place the film at a distance fd/D on either side of the true focal plane without appreciable degradation of the image. For example, if the aperture is D = f/6.3, and you are prepared to tolerate an out-of-focus diameter d = 0.1 mm, the depth of focus will be ±6.3d or ±0.63 mm. On the other hand if you "stop down" to D = f/22, your depth of focus will be 2.2 mm. Note that we have not been considering here the effect of spherical aberration, but of course this, too, increases with aperture, as well as merely the "out-of-focus" effect.

Notice that the tangent of the semi angle of the converging cone is R/f, or $D/2f$. For apertures of f/6.3 and f/22, the semi angles are $4^{\circ}.5$ and $2^{\circ}.6$ respectively. This may give some comfort to those readers who have been uncomfortable with our assumption that angles are small. I have not been able to draw the lens and mirror drawings in these chapters with realistically small angles, because the drawings would be too cramped. I hope you will understand this shortcoming; you are welcome to try yourself!

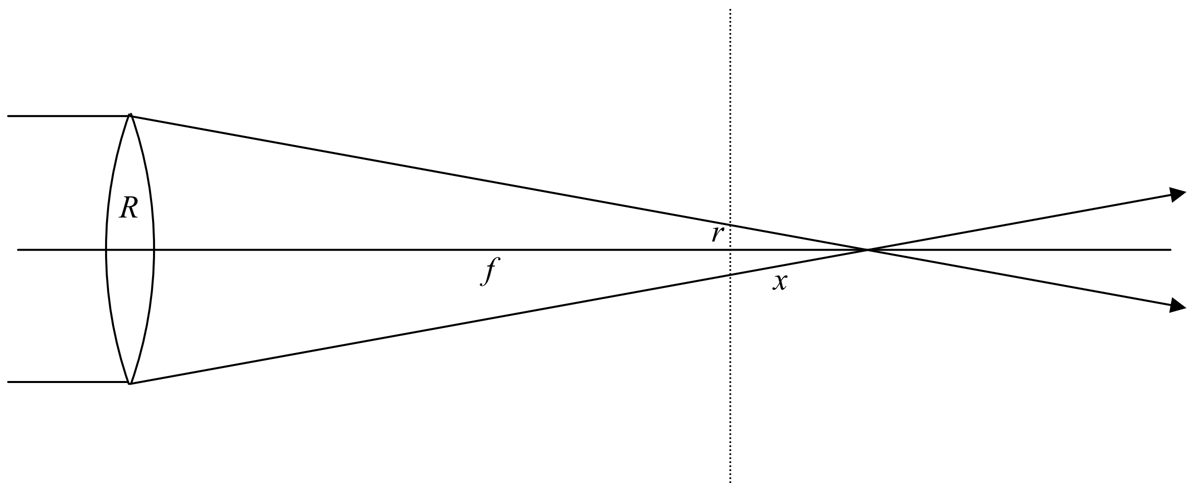

FIGURE III.4

Depth of focus is not the same thing as *depth of field.* Suppose we want to photograph an object at a distance p from the camera lens, and that we are prepared to tolerate an out-of-focus "image" of diameter up to d, or radius r. Any object at a distance within the range $p \pm \Delta p$ may satisfy this, and we now want to find Δp. Figure III.5 shows, with full lines, light from an object at distance p coming to a focus at a distance q, and with dashed lines, light from an object at a distance Δp closer to the lens coming to a focus at a distance Δq further from the lens. The position of the film is indicated by the dotted line, and the radius of the out-of-focus dashed "image" is r.

We have $$\frac{1}{q} = -\frac{1}{p} + \frac{1}{f}, \qquad 3.5.1$$

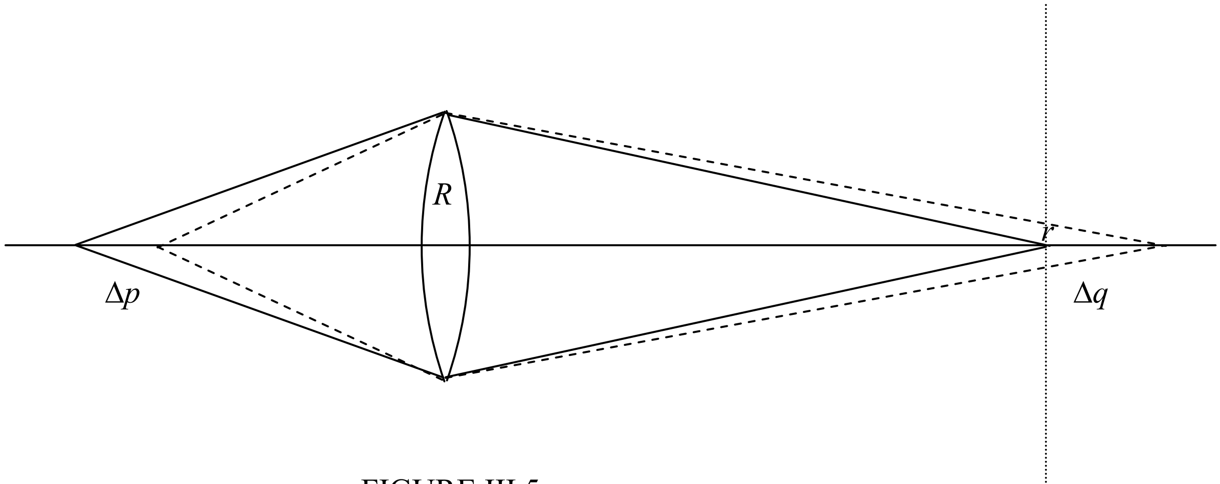

FIGURE III.5

so that
$$q = \frac{pf}{p-f} \tag{3.5.2}$$

and, without regard to sign

$$\Delta q = -\left(\frac{f}{p-f}\right)^2 \Delta p. \tag{3.5.3}$$

From similar triangles we see that

$$\frac{R}{q+\Delta q} = \frac{r}{\Delta q}. \tag{3.5.4}$$

Elimination of q and Δq results in

$$\Delta p = \frac{pr(p-f)}{f(R-r)}, \tag{3.5.5}$$

or, in terms of diameters rather than radii,

$$\Delta p = \frac{pd(p-f)}{f(D-d)}. \tag{3.5.6}$$

For example, suppose the focal length is $f = 25$ cm and you want to photograph an object at a distance of $p = 400$ cm. You are prepared to regard an out-of-focus "image" tolerable if its diameter is no larger than $d = 0.1$ mm. If the aperture is $D = f/6.3$, you can photograph objects in the range (400 ± 15) cm, whereas if you "stop down" to $D = f/22$, you can photograph objects in the range (400 ± 53) cm.

To the approximation that $d \ll D$ and $f \ll p$, equation 3.5.6 becomes

$$\Delta p \approx \frac{p^2 d}{f\,D}. \qquad 3.5.7$$

3.6 *The Telescope*

As mentioned in section 3.1,our purpose here is not to describe at length of the details of modern telescope design, but just to give the basic principles of a simple telescope at a level needed to answer first-year examination questions and not necessarily to describe a telescope that one might actually be able to see anything through! An advanced astronomy student wanting details of real telescopes will have to search elsewhere. That said, the basic principles of a simple telescope still apply to real telescopes. Figure III.6, then, illustrates a telescope in its simplest form. Because of the difficulty of drawing diagrams with small angles, the telescope looks very stubby compared with a real one. To make a more realistic drawing, most of the angles should be less than about one degree.

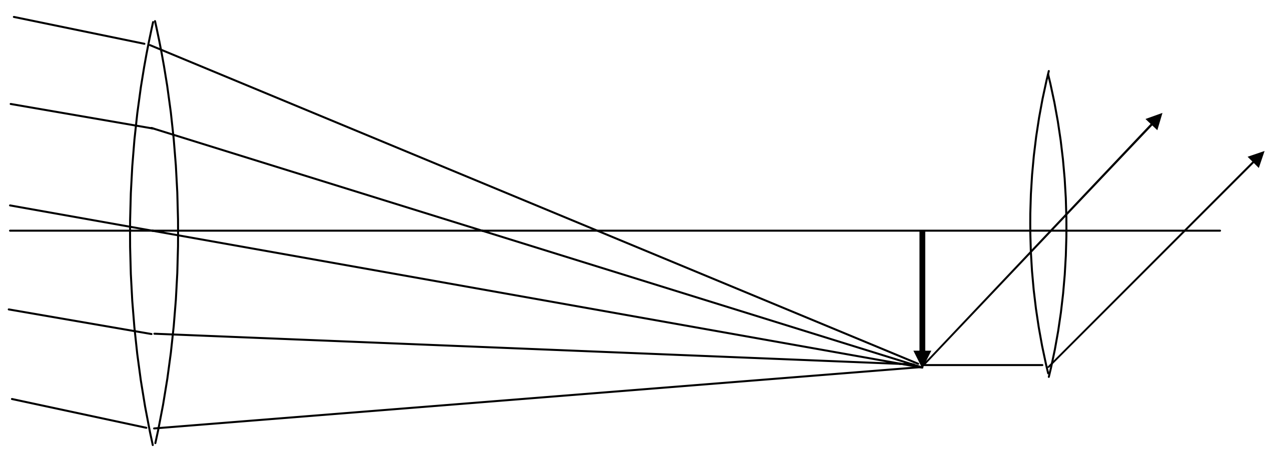

FIGURE III.6

We see at the left hand side of the figure a parallel beam of light coming in from a distant object off-axis. The first lens that it encounters is the *object glass*. Its function is to produce a real image in its focal plane, and the distance between the object glass and this *primary image* is f_1, the focal length of the object glass. In a real bird-watching telescope, the object glass in reality is a crown-flint achromatic doublet that brings all colours to almost the same focus. In a large astronomical telescope, instead of a lens, the primary image is formed by a large concave mirror that is often paraboloidal rather than spherical in shape.

If the telescope is an astronomical telescope intended for photography, that is all there is to it. There is no second lens. The primary image falls directly on to a photographic plate or film or CCD. Let is suppose that we are looking at the Moon, whose angular radius is about a quarter of a degree and whose actual linear radius is about 1740 km. The distance of the Moon is about 384,000 km. We'll suppose that the telescope is pointed straight at the centre of the Moon, and that the beam of light coming in from the left of figure III.6 is coming from the upper limb of the Moon. The image of the upper limb of the Moon is the tip of the thick arrow. The radius of the image of the Moon (i.e. the length of the thick arrow) is $f_1 \tan \frac{1}{4}^\circ$. We'll suppose that we are using a fairly large telescope, with a focal length of ten metres. The radius of the primary image is then 4.4 cm, whereas the radius of the object (the Moon) is 1740 km. Thus the function of the object glass is to produce an image that is very, very, very much smaller that the object. The linear magnification is 4.4/174,000,000 or 2.5×10^{-8}. This is also equal to image distance divided by object distance, which is 10/384,000,000. And you thought that a telescope magnifies!

However, rather than using the telescope for photography, we want to "look through" the telescope. We don't want a photographic plate at the position of the real image. Instead, all we have to do is to look at the real image with a magnifying glass, and that is what the second lens in figure III.6 is. This second lens, which is just a magnifying glass (which, we have seen in section 3.3, doesn't magnify either!) is called the *eyepiece*. As is usual with a magnifying glass, the thing we are looking at (which is the primary image produced by the object glass, but which serves as an object for the eyepiece) is placed in the focal plane of the eyepiece, so that parallel light emerges from the eyepiece. As explained in section 3.3 you don't have to think about this – your ciliary muscles are most relaxed when the eye is ready to received parallel light. The eyepiece of a telescope can usually be moved in and out until the image appears sharp to your relaxed eye. Thus the primary image is in the focal plane of the object glass and also of the eyepiece, and the distance between object glass and eyepiece is $f_1 + f_2$, where f_1 and f_2 are the focal lengths of object glass and eyepiece respectively. I have drawn the usual two rays from the primary image (which is the object for the eyepiece), namely one that goes straight through the centre of the lens, and one parallel to the axis, which subsequently passes through the focal point of the eyepiece.

Figure III.7 is figure III.6 redrawn with all but two rays removed, namely the ray that passes through the centre of the object glass and the ray that passes through the centre of the eyepiece.

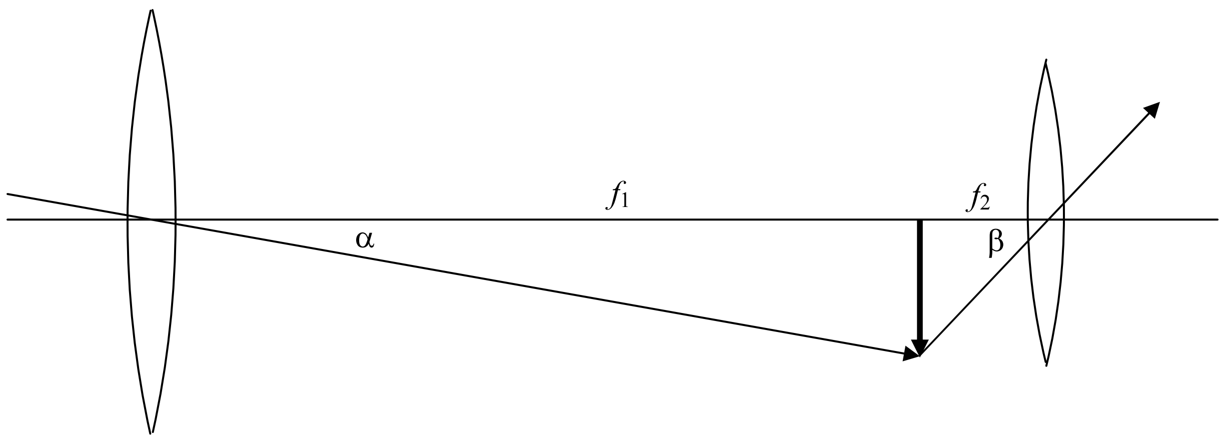

FIGURE III.7

Although, as we have seen, the linear size of the primary image is very much smaller than (i.e. centimetres rather than thousands of kilometres!) the object, what counts when we are looking through a telescope is the *angular magnification*, which is the ratio of the angular size of the image to the angular size of the object – that is the ratio β/α. And since, as usual, we are dealing with small angles (the angular diameter of the Moon is only about half a degree) – even though it is difficult to draw a realistic diagram with such small angles – this ratio is just equal to f_1/f_2. Note that the *definition* of the angular magnification is the ratio of the angular size of the image to the angular size of the object (and this time we don't add "when the object is at the near point"!), while f_1/f_2 is how we can *calculate* it. Thus, if you are asked what is meant by the angular magnification of a telescope, and you say "f_1/f_2" you will get nought out of ten – and deservedly so.

In any case, for large magnification, you need an object glass of long focal length and an eyepiece of short focal length. Generally you have a choice of several eyepieces to choose from.

It should be pointed out that magnification is not the most important attribute of a large astronomical telescope. Large astronomical telescopes have large primary mirrors mainly to collect as much light as possible.

Exercise. A telescope is used with an eyepiece that magnifies 8 times. The angular magnification of the telescope when used with this eyepiece is 200. What is the distance between object glass and eyepiece? *Answer*: 628.125 cm.

One thing is odd about the "telescope" described so far – the image is upside down! In fact for astronomical purposes this doesn't matter at all, and there is nothing "wrong". For a telescopes designed for terrestrial use, however, such as for bird-watching, we want the image to be the right way up. In older telescopes this was done with two additional lenses; in modern telescopes the image is reversed with additional prisms.

The astute reader may notice that there is something else wrong with figures III.6 and 7. The object glass produces a real primary image, and then we examine that real primary image with a magnifying glass. But look at the ray that goes from the tip of the primary image through the centre of the eyepiece. Where did it come from? It doesn't seem ever to have passed through the object glass! Part of the answer to this is that angles in the drawings are grossly exaggerated (it is too difficult to draw diagrams with realistically small angles), and that if the angles were correctly drawn, this rogue ray would indeed be seen to have passed through the object glass. But this is only part of the answer, and a telescope with just the two lenses shown would have a very small field of view.

In practice an *eyepiece* consists of two lenses separated by a short distance. These two lenses are called the *field lens* and an *eye lens*. In one arrangement the field lens coincides with the primary image – i.e. the primary image formed by the object glass falls exactly on the field lens. The field lens does not affect the magnification at all; it merely serves to bend some of the light from the object glass into the eye lens. The rogue ray that we have called attention to has been bent towards the eye lens by the field lens. This arrangement would work, although one problem that would arise is that bits of dust on the surface of the field lens would be in sharp focus when viewed with the eye lens. Thus the field lens is often arranged so as not to coincide exactly with the primary image. Eyepiece design could easily occupy an entire chapter, and it is not uncommon for a good eyepiece to have six or more components; we just mention this particular problem to illustrate some of the points to be considered in optical design.

Let us return to our simple telescope of just two lenses. Let us look at things from the point of view of the eyepiece (which, in our simple telescope, consists of just the eye lens). If we now regard the object glass as an *object*, we can understand that the eyepiece will produce a real image of this "object". See figure III.8.

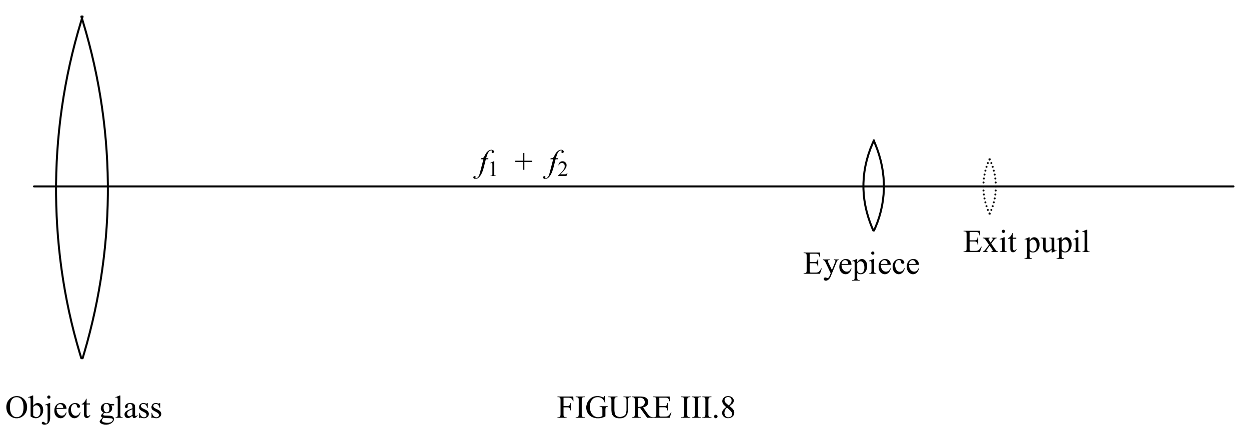

FIGURE III.8

The real image of the object glass produced by the eyepiece is called the *exit pupil* of the telescope, and the object glass is the *entrance pupil* of the telescope. All light that passes through the entrance pupil also passes through the exit pupil. You can easily see the exit pupil a few millimetres from the eyepiece if you hold a pair of binoculars in front of you at arm's length. The notation such as "10 $\times$ 50", which you see on a pair of binoculars means that the angular magnification is 10 and the diameter of the object glass is 50 mm. If you look at the exit pupils of a pair of binoculars that you are considering buying, make sure that they are *circular* and not square. If they are square, some of the light that passed through the entrance pupil is being obstructed, probably by inadequate prisms inside the binoculars, and you are not getting your full 50 millimetres' worth. The size of the exit pupil should be approximately equal to the size of the entrance pupil of your eye. This is about 4mm in sunlight and about 7 mm at night – so you have to consider whether you are going to be using the binoculars mainly for birdwatching or mainly for stargazing.

Just where is the exit pupil, and how big is it? "Where?" is just as important a question as "how big?" – the distance between the eyepiece and the exit pupil is the *eye relief*. You want this distance to be small if you do not wear glasses. If you are merely myopic or hypermetropic, there is no need for you to wear your glasses when using binoculars or a telescope – you can merely adjust the focus of the telescope. If you wear glasses to correct for astigmatism, however, you will still need your glasses when using the binoculars or telescope, so you will need a larger eye relief.

To find the eye relief, or distance of the exit pupil from the eye lens, recall that the distance between object glass and eyepiece is $f_1 + f_2$, and the focal length of the eyepiece is f_2. The eye relief is therefore given by $\frac{1}{q} = -\frac{1}{f_1 + f_2} + \frac{1}{f_2}$, or $q = \frac{f_2(f_1 + f_2)}{f_1}$. The ratio of the size of the entrance pupil to the size of the exit pupil is equal to the ratio of their distances from the eyepiece. This is just f_1/f_2, which is the angular magnification of the telescope. Thus the diameter of the exit pupil of a pair of 10 $\times$ 50 binoculars is 5 mm – just divide the diameter of the object glass by the magnification.

3.7 *The Microscope*

The front lens of a microscope is generally called the "objective" lens, rather than the "object glass". In contrast to the telescope, the objective is a small lens with a short focal length. The object is placed *just outside* the focal point of the objective, and a magnified real inverted primary image is formed quite some distance away. This is examined with an eyepiece in the same way that the primary image formed in a telescope is examined with an eyepiece. See figure III.9. As discussed for the telescope eyepiece, the eyepiece in reality has a second lens (the "field lens"), which I have not drawn, which almost (but not quite) coincides with the primary in order to bend that vexing ray towards the centre of the eye lens. The primary image is in the focal plane of the eyepiece, but (unlike for the telescope) it is not in the focal plane of the objective,

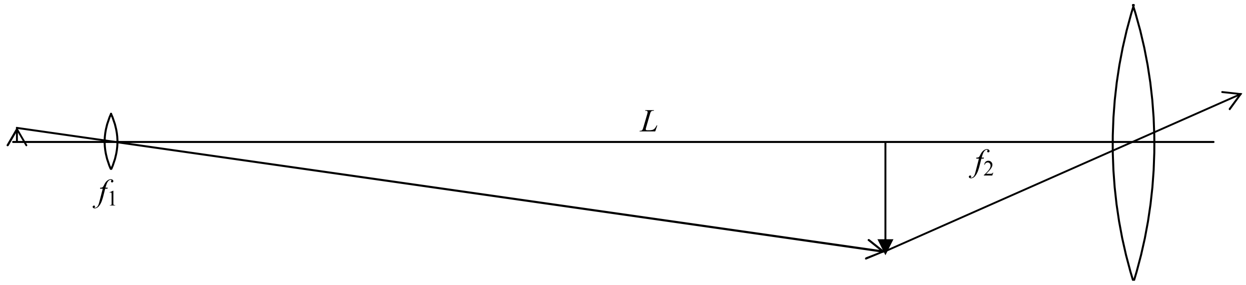

FIGURE III.9

Everyone knows how to calculate the angular magnification produced by a magnifying glass (D/f) and by a telescope (f_1/f_2). A microscope isn't quite so easy, which is why, in an exam, you will be asked for the magnification of a microscope rather than of a magnifying glass or a telescope. When you are focussing a telescope, you pull the eyepiece in and out until the image appears in focus for your relaxed eye. When you are focussing a microscope, however, rather than moving just the eyepiece, you move the whole microscope tube up and down, in such a manner that the distance L between the two lenses is constant. What we need, then, is to find the magnification in terms of the two focal lengths and the distance L between the lenses.

Recall the way a microscope works. First, the objective produces a magnified real image of the object. Then you look at this primary image with an eyepiece. The overall magnification, then, is the *product* of the *linear magnification produced by the objective* and the *angular magnification produced by the eyepiece*. We shall address ourselves to these two in turn.

To find the linear magnification produced by the objective, we need to know the object and image distances. The image distance is just $L - f_2$, and, since the focal length of the objective is f_1, it doesn't take us a moment to find that the object distance is $\frac{f_1(L-f_2)}{L-f_1-f_2}$. Therefore the linear magnification produced by the objective is $\frac{L-f_1-f_2}{f_1}$. And the angular magnification produced by the eyepiece is just D/f_2, where D is the distance to the near point (25 cm). Thus the overall angular magnification is $\frac{L-f_1-f_2}{f_1} \times \frac{D}{f_2} . \frac{L-f_1-f_2}{f_1}$. Voilà! It's easy!